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HEATING OF A FILM FLOWING OVER THE SURFACE OF A ROTATING HEAT TRANSFER DEVICE

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The authors have solved the problem of a liquid flow over the surface of a rotating conic heat-transfer device with thermal boundary conditions of the second kind with account of heat losses to the vapor-gas phase. Relations have been obtained for calculation of the mean mass temperature, the Nusselt number, and other characteristics of the heating process of the film.

In the chemical industry use is made of centrifugal film heat exchangers, whose main component is rotating heated (cooled) heat transfer devices in form of discs and cones, over the surface of which the treated liquid is spread under the action of centrifugal forces [1]. Because of the presence of a thin film, these apparatuses are very efficient. The heating of a nonisothermal viscous flow in a field of centrifugal forces was considered most thoroughly in [2]. However, the problem was solved with the assumption of smallness of the inertial terms in the equation of motion. So, it is interesting to include the effect of the inertial terms on the flow hydrodynamics and, consequently, on the heat-transfer process.

We will consider the axisymmetric flow of a thin film of viscous liquid over the surface of a conic heat transfer apparatus (Fig. 1). Motion of the liquid is described in the coordinate system l, φ, which is rigidly coupled with the rotating cone. The flow is steady and laminar. The temperature T_0 of the liquid fed to the cone is constant and equal to the temperature of the ambient gas. The heat flux over the conic surface is constant in time and over the radius and equal to q_w. The effect of the temperature nonuniformity of the flow on its hydrodynamics is exhibited through the appropriate deformation rates; therefore, the liquid viscosity μ is introduced as a function of the temperature T and expressed as an expansion:

$$\mu_0/\mu = 1 + a_1 (T - T_0) + a_2 (T - T_0)^2 + \dots \tag{1}$$

The number of terms in expansion (1) depends on the required calculation accuracy. In view of the fact that the absolute heat flux for heating of the film flow over the conic surface is not high, in engineering calculations a linear approximation in relation (1) is quite sufficient; the preservation of terms of the second and higher orders does not lead to mathematical complications, but the resultant analytical formulas become more cumbersome.

Assumptions conventional for thin-film problems lead to the following system of equations describing the flow and heating of the film:

$$\rho (\omega^2 l \sin \eta - g \cos \eta) + \frac{\partial}{\partial z} \left| \mu \frac{\partial v_l}{\partial z} \right| = v_l \frac{\partial v_l}{\partial l} + v_z \frac{\partial v_l}{\partial z}; \tag{2}$$

$$v_l \frac{\partial T}{\partial l} + v_z \frac{\partial T}{\partial z} = a \frac{\partial^2 T}{\partial z^2}; \tag{3}$$

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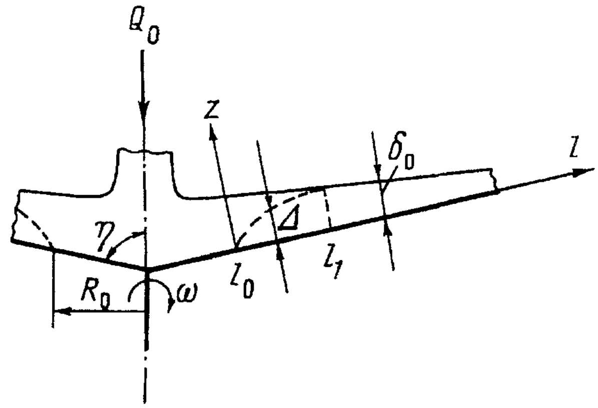

Fig. 1. Schematic drawing of the flow.

$$\frac{\partial v_l}{\partial l} + \frac{v_l}{l} + \frac{\partial v_z}{\partial z} = 0 . \tag{4}$$

If heating of the film is considered from the point of view of development of a thermal boundary layer (TBL) in the film, three zones can be distinguished reasonably in the flow: the initial thermal section, the region of heating of the film, and the region of steady-state heat transfer. Now, we will consider heat transfer processes in each of the zones.

Initial Thermal Section. Within the initial thermal section in the wall region of the liquid film a TBL appears and expands down the flow. At the lower boundary of the TBL the tempearture of the liquid is equal to the temperature of the heated cone T_0, which varies over the radius and depends on the value of the heat flux q_w. At the upper boundary, $T = T_0$. If its is assumed that the heat flux enters the liquid starting from a certain radius R_0, the TBL arises at the boundary R_0. The other boundary of the initial thermal section will certainly be at the radius R_1 at which the thickness of the TBL is equal to that of the film.

It seems necessary that the principles for choosing the initial value of R_0 be strictly defined. Hydrodynamically, in a liquid spreading over the surface of a rotating cone the following zones can be distinguished: a central zone, in which the liquid flow contacts the cone and convective forces exceed substantially centrifugal forces; an intermediate zone, in which these forces are of the same order of magnitude; and a zone of a film flow, in which centrifugal forces exceed substantially inertia forces, the development of the velocity boundary layer has been completed, and the flow has become a laminar film flow. The coordinate of the initial zone of the film flow R_0' can be defined by the formula [3]:

$$R_0' = \left(\frac{\rho Q_0^2}{4\pi\mu_0\omega} \right)^{1/4} . \tag{5}$$

In solution of the problem it is assumed that the condition $R_0 \geq R_0'$ is satisfied.

In view of the above statements, the heat transfer process satisfies the following boundary and initial conditions:

$$z = 0: \ \tau = \tau_0 , \quad v_l = v_z , \quad q_w = -\lambda \frac{\partial T}{\partial z} = \text{const} ;$$

$$z = \Delta: \ v_l = v_l^* , \quad \frac{\partial v_l}{\partial z} = \frac{\partial v_l^*}{\partial z} , \quad T = T_0 , \quad -\lambda \frac{\partial T}{\partial z} = 0 ;$$

$$z = \delta_0: \ \tau = 0 , \quad T = T_0 , \quad l = l_0: \ \Delta = 0 ; \quad l = l_1 ; \quad \Delta = \delta_0 . \tag{6}$$

As the unknown parameters change monotonically, for solution of the problem we use averaging by integration of Eqs. (2) and (3) over the coordinate z from 0 to δ_0 [4]. Using continuity Eq. (4), Eqs. (2) and (3) can be transformed to:

$$\frac{1}{l}\frac{d}{dl}l\int_0^{\Delta} v_l^2 dz + \frac{1}{l}\frac{d}{dl}l\int_{\Delta}^{\delta_0} v_l^{*2} dz = F_l\delta_0 - \frac{\tau_0}{\rho}; \tag{7}$$

$$\frac{1}{l}\frac{d}{dl}l\int_0^{\Delta} Tv_l dz = -a\left(\frac{\partial T}{\partial z}\right)_{z=0}. \tag{8}$$

Assuming that the temperature of the liquid changes smoothly over the coordinate z within the TBL with chosen boundary conditions (6), it is possible to find the temperature profile:

$$\theta = \frac{s}{2}\left(1 - \frac{\delta}{s}\right)^2. \tag{9}$$

Relation (9) is expressed in dimensionless form: $\theta = (T - T_0)/(q_w\delta_0/\lambda)$; $(s, \delta) = (\Delta, z)/\delta_0$. Then expansion (1) expressed in terms of the parameters of the TBL is written as

$$\frac{\mu_0}{\mu} = 1 + K_1\theta + K_2\theta^2 + \dots, \quad K_i = a_i\left(\frac{q_w\delta_0}{\lambda}\right)^i. \tag{10}$$

As was shown in [5], the shear stress can be approximated as a single-valued function of the film thickness, whose expansion coefficients are found from boundary condtiions (6). If we take only the first two terms of the expansion, we can write:

$$\frac{\partial v_l}{\partial \delta} = \frac{\tau_0\delta_0}{\mu}(1 - \delta). \tag{11}$$

Integration of Eq. (11) results in the following expressions for the meridional velocity within and outside the TBL:

$$v_l = \frac{\tau_0\delta_0}{\mu_0}\left\{\delta - \frac{\delta^2}{2} + \frac{K_1 s}{2}\left[\delta - \delta^2\left(\frac{1}{s} + \frac{1}{2}\right) + \frac{\delta^3}{3s}\left(\frac{1}{s} + 2\right) - \frac{\delta^4}{4s^2}\right]\right\}; \tag{12}$$

$$v_l^* = \frac{\tau_0\delta_0}{\mu_0}\left[\delta - \frac{\delta^2}{2} + \frac{K_1}{24}(4s - s^2)\right]. \tag{13}$$

The unknown τ_0 is found from the condition of a constant flow rate in the initial thermal section:

$$Q_0 = 2\pi l\delta_0 \sin\eta\left[\int_0^s v_l d\delta + \int_s^1 v_l^* d\delta\right]. \tag{14}$$

Eventually, we have

$$\tau_0 = \frac{3\mu_0 Q_0}{2\pi l\delta_0^2 \sin\eta\left[1 + \frac{K_1 s^2}{20}(10 - 5s + s^2)\right]}. \tag{15}$$

Analysis of relation (15) has shown that, first, the value of the shear stress on the conic wall is lower in the case of a nonisothermal flow as compared with an isothermal flow and, second, it decreases as l and s rise.

Substitution of (9), (12), and (13) and integration transform Eqs. (7) and (8) to homogeneous equations of the form

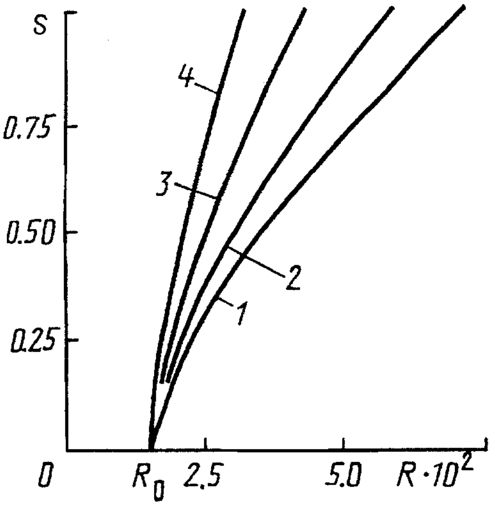

Fig. 2. Variation of the thickness of the TBL over the radius (q_w = 7.5 kW/m^2): 1) Q_0 = 46.3·10^{-6} m^3/sec; ω = 31.4 sec^{-1}; C = 65%; 2) 31.8·10^{-6}; 31.4; 65; 3) 31.8·10^{-6}; 31.4; 25; 4) 31.8·10^{-6}; 125.6; 25. R, m.

$$A_1 \frac{d\delta_0}{dl} + B_1 \frac{ds}{dl} = C_1 ; \tag{16}$$

$$A_2 \frac{d\delta_0}{dl} + B_2 \frac{ds}{dl} = C_2 , \tag{17}$$

$$(A_{1,2} ;\ B_{1,2} ;\ C_{1,2}) = f(\delta_0 ,\ s ,\ Q_0 ,\ l ,\ \mu_0 ,\ \eta ,\ \omega) ,$$

These equations constitute a system for finding the unknown parameters δ_0 and z in terms of which hydrodynamic and heat transfer characteristics of the flow are expressed. For example, for calculation of the most important integral characteristic of the process, the flow rate-average temperature $\bar{\theta}$, following the formula is obtained:

$$\bar{\theta} = \frac{\pi l \delta_0^2 \tau_0 s^3 \sin \eta}{\mu_0 Q_0} \left[\frac{1}{12} - \frac{s}{60} + \frac{K_1 s}{36} \left(1 - \frac{s}{7} \right) \right] . \tag{18}$$

The local heat transfer coefficient expressed in terms of the temperature gradient on the conic surface is found from the expression

$$\alpha = \frac{2\lambda}{\delta_0 (s - 2\bar{\theta})} . \tag{19}$$

In the initial thermal section the dimensionless heat transfer coefficient (the Nusselt number) is found from the relation

$$\mathrm{Nu} = \frac{4\alpha s \delta_0}{\lambda} = \frac{8s}{s - 2\bar{\theta}} . \tag{20}$$

As can be seen from Eq. (20), the relations governing heat transfer in the TBL depend mainly on the thickness of the thermal boundary layer s. The dynamics of TBL development is shown in Fig. 2. In the initial zone one can see a sharp increase in the thickness of the TBL. In this case the liquid flow rate, the angular velocity of the heat transfer device, and the preset heat flux density have a substantial effect on the heat transfer process.

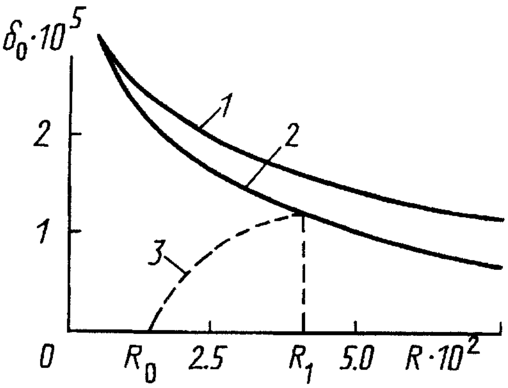

Fig. 3. Variation of the film thickness and TBL over the radius for a glycerin solution ($Q_0 = 46.3 \cdot 10^{-6}$ m^3/sec; $\omega = 31.4$ sec^{-1}; $C = 25\%$: 1) $\delta_0 = f(R)$, $q_w = 0$; 2) $\delta_0 = f(R)$, $q_w = 188.8$ kW/m^2; 3) $\Delta = f(R)$, $q_w = 188.8$ kW/m^2. δ_0, m.

2. The Region of Film Heating. In this region the liquid is involved in heat transfer throughout the film thickness. The temperature of the film surface increases gradually and heat losses to the vapor-gas medium grow simultaneously due to (a) conduction heat transfer from the liquid-gas interface q_c and (b) evaporation of some of the liquid q_e. Thus, the heat flux from the surface of the film will be equal to

$$q = q_c + q_e = \alpha_{\text{liq.g}} (T_s - T_g) + \frac{\beta r}{R^* T_s} (p_{\text{sat}} - p_g) . \tag{21}$$

Using the analogy method, expresson (21) can be transformed to the form [2]

$$q = \lambda_g \sqrt{(\omega/\nu_g) \sin \eta} \; t'(0) \left[T_s - T_g + \frac{r}{R^* C_{pg} \rho_g T_s} \left(\frac{D}{a_g} \right)^{1-n} (p_{\text{sat}} - p_g) \right] . \tag{22}$$

Since the saturated vapor pressure p_{sat} is a single-valued function of temperature, the heat flux q will be, as a whole, only a function of the temperature of the film T_s.

At the beginning of the second region ($l = l_1$), the temperature of the film surface is T_0. The heating region ends at the site where the heat fluxes (the flux entering the film and the flux scattered by the film to the vapor-gas medium) become equal. At that moment the film stops receiving heat from the heater and a linear temperature prodile develops in it due to Fourier's law.

Thus, for the second region the boundary conditions have the form:

$$\delta = 0 : \; v_l = v_z = 0 , \quad \frac{\partial \theta}{\partial \delta} = -1 ;$$

$$\delta = 1 : \; \frac{\partial v_l}{\partial \delta} = 0 , \quad \theta = \theta_s , \quad \frac{\partial \theta}{\partial \delta} = -\frac{q}{q_w} ; \tag{23}$$

$$l = l_1 : \; \theta_s = 0 ; \quad l = l_2 : \; q = q_w .$$

Methods of solution similar to those used for the region in which the TBL is developed can be used to find the temperature distribution across the film and the meridional component of the velocity:

$$\theta = 1 + \theta_s - \delta - N (1 - \delta^2) ; \tag{24}$$

$$v_l = \frac{\tau_0 \delta_0}{\mu_0} \left\{ [1 + K_1 (1 + \theta_s - N)] \delta - [1 + K_1 (2 + \theta_s - N)] \frac{\delta^2}{2} + K_1 (N + 1) \frac{\delta^3}{3} - K_1 N \frac{\delta^4}{4} \right\} . \tag{25}$$

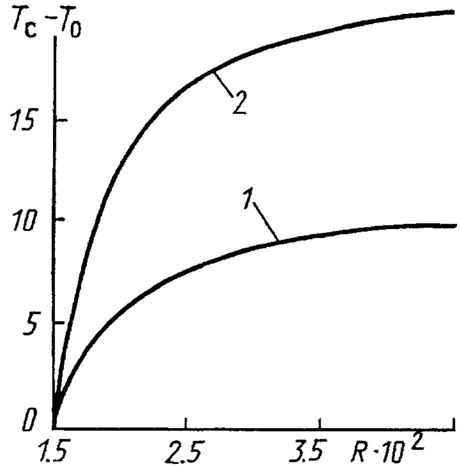

Fig. 4. Dynamics of heating of the wall of the heat transfer device ($Q_0 = 46.3 \cdot 10^{-6}$ m^3/sec, $\omega = 31.4$ sec^{-1}; $C = 25\%$: 1) $q_w = 47.2$ kW/m^2; 2) 188.8. $T_c - T_0$, °C.

Here $N = 0.5(1 - q/q_w)$.

For calculation of the shear stress on the conic wall in the second region, the following formula is obtained:

$$\tau_0 = \frac{3\mu_0 Q_0}{2\pi l \delta_0^2 \sin\eta \left[1 + \frac{K_1}{20}(15 + 20\theta_s - 18N)\right]}. \tag{26}$$

In the second region the dimensionless heat transfer coefficient (the Nusselt number) is defined by

$$\mathrm{Nu} = \frac{4\alpha\delta_0}{\lambda} = \frac{80 + 4K_1(15 + 20\theta_s - 18N)}{12.5 - 9N - K_1\left(-9 - 12.5\theta_s + \frac{52}{3}N + 9N\theta_s - \frac{82}{189}N^2\right)}. \tag{27}$$

In expression (27) the difference between the local surface temperature of the heat transfer cone and the mean mass temperature of the liquid film in the cross-section considered is also used as a temperature head in determination of α.

Since in the second region heating of the surface of the liquid film is only started, it can be assumed that when considerable heat is transferred to the vapor-gas phase due to evaporation, the mass loss remains insignificant. Together with the assumptions made earlier here, this assumption allows us to transform Eqs. (2)-(4), which are also valid in the second region, to the form:

$$\frac{1}{l}\frac{d}{dl} l \int_0^{\delta_0} v_l^2 dz = F_l \delta_0 - \frac{\tau_0}{\rho}; \tag{28}$$

$$\frac{1}{l}\frac{d}{dl} l \int_0^{\delta_0} T v_l dz = \frac{a}{\lambda}(q_w - q). \tag{29}$$

Substitution of relations (24) and (25) also transforms Eqs. (28) and (29) to a system of differential equations (16) and (17). In this case the unknown quantities are δ_0 and θ_s and in the second region all the other parameters of the film flow and heating are expressed in terms of them.

Having the analytical relations, obtained it is possible to estimate the dynamics and behavior of all the other parameters. For example, at the end of the second zone the Nusselt number tends asymptotically to 6.4. It should be noted that in gravity film flows Nu is also 6.4 [6].

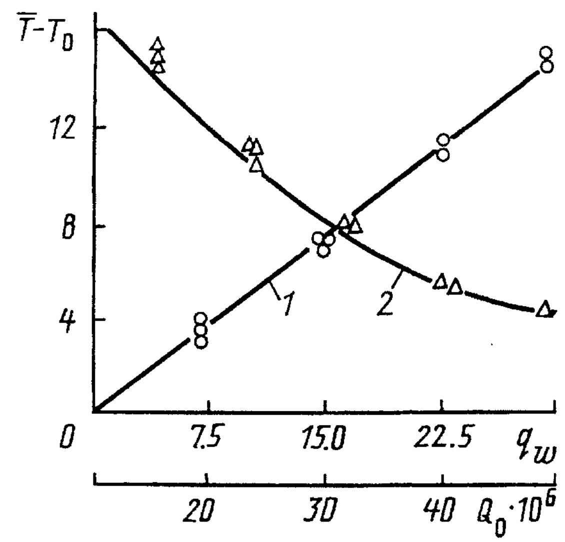

Fig. 5. Plot of the mean mass temperature of the liquid versus q_w and Q_0 ($\omega = 31.4$ sec^{-1}; $C = 25\%$, $R = 0.1$ m): 1) $(\bar{T} - T_0) = f(g_w)$, $Q_0 = 17.3 \cdot 10^{-6}$ m^3/sec; 2) $(\bar{T} - T_0) = f(Q_0)$, $q_w = 30$ kW/m^2; Q_0, m^3/sec.

3. Zone of Steady-State Heat Transfer. In this zone the whole liquid film is heated, the temperature distribution is linear, the surface temperature is constant, and the temperature drop between the conic and film surfaces is

$$T_c - T_s = q_w \delta_0 / \lambda . \tag{30}$$

In the third region intense mass transfer occurs due to evaporation, which results in loss of mass of the flowing liquid, and appropriate choice of the operation parameters can lead to complete drying of the entire film. In view of the balance considerations given in [2], one can estimate the radius at which the film will be dried out.

Calculations carried out with the relations given here earlier have shown that inclusion of the inertial terms in Eq. (2) leads to a 3–8% decrease in the film thickness, depending on the operation parameters, which, in turn, affect the dynamics of the process of film heating. It has been found by calculations and experiments that heating of the cone decreases substantially the thickness of the liquid film. In particular, at the end of the TBL at $s = 1$ the difference between the values of δ_0 for isothermal and nonisothermal flows can be 15% (Fig. 3). In the studies, aqueous solutions of glycerin at different concentrations were used as model liquids, whose physical and thermal properties are given in [7]. It should be noted that this difference ibcreased with an increase in the heat flux and concentration. For heat-transfer problems with boundary conditions of the second kind (q_w = const), the surface temperature of the heat transfer device is also an unknown value. The behavior of the surface temperature of the heat exchanger over the radius is shown in Fig. 4, and the effect of the heat-flux density on the mean mass temperature is presented in Fig. 5.

Experimental studies on heating of aqueous solutions of glycerin on a flat disk with a radius of 0.2 m heated by electric current have confirmed the validity of the obtained relations. For example, the difference between the experimental and calculated mean mass temperatures is 10–20%. Because of this, the relations obtained in this work can be recommended for the design of centrifugal heat and mass transfer apparatus.

NOTATION

v_l, v_z, meridional and normal velocity components, m/sec; ω, angular velocity, sec^{-1}; μ, viscosity, Pa·sec; a thermal diffusivity, m^2/sec; ρ, liquid density , m^3/sec; g, gravity acceleration, m^2/sec; τ, shear stress, Pa; q, heat flux density, W/m^2; λ, thermal conductivity, W/(m·K); Δ, thickness of TBL, m; s, dimensionless thickness of TBL; Q_0, liquid flow rate, m^3/sec; δ_0, thickness of the liquid film, m; r, specific evaporation rate, J/kg; βv, mass-transfer coefficient, m/sec; α, heat transfer coefficient, W/(m^2·K); R^*, gas constant, J/(kg·K); c_p, specific heat, J/(kg·K); p_{sat}, saturated vapor pressure, Pa; ν, kinematic viscosity, m^2/sec; D, diffusion coefficient, m^2/sec; F_l, projection of the mass force onto the coordinate axis l, $-m^2$/sec. Subscripts: g, gas medium; s, surface of the liquid film; 0, initial value of the parameter.

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